Static and Dynamic Behavior of Quartz Resonators

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Abstract–The frequency-temperature (f-T) behavior of a crystal resonator depends upon temperature, and its spatial and temporal gradients. For quasi-isothermal changes the static f-T curve can be used to determine frequency shifts that occur, e.g., in oven-controlled units. The frequency then depends upon the parameters of the static f-T curve, the temperature range over which the oven cycles, and upon the oven setting point. The maximum frequency excursion has been computed for the AT and SC cuts of quartz in terms of these parameters as a function of the orientation angle. When thermal-transient-compensated cuts are not utilized, oven cyclings or other temperature perturbations introduce an additional nonnegligible component of the frequency shift. This effect is quantified by means of a simple mathematical model. The model is capable of predicting the thermal transient effects for AT cuts appearing in the literature. Simulations, using the model, disclose that sinusoidal temperature variations with periods of hours can readily lead to frequency instabilities much larger than would be expected using the static f-T curve for the AT cut. This effect should be greatly *diminished in the vicinity of the SC* cut.

Introduction

IN THEIR sixty years quartz resonators have shown themselves capable of dramatic improvements in frequency stability: on the average, one order of magnitude per decade. Because of a long history compared with other solid-state devices, it is easy to look upon this component as the product of a mature technology and to presume that the leveling-off plateau has been reached; that the last bit of stability has been, or shortly is to be, wrung out of the venerable vibrator.

Such a conclusion would be entirely wrong. Prospects are exceedingly good that the near-term future will see a further hundredfold improvement in the stability of high precision oscillators, brought about by using doubly rotated quartz resonators of special orientations. This development fortunately coincides with stringent frequency control requirements newly imposed by the latest generations of communication systems and systems for navigation and position location.

In order to identify an important present contribution to frequency instability and to separate out its effects, it is convenient to discuss first the static frequency-temperature (f-T) characteristic of a quartz resonator. Static f-T curves are of interest in their own right in connection with crystals operating without ovens over broad temperature ranges, such as in temperature compensated crystal oscillators [2]. Maximum frequency deviations, obtained from the static f-T curves, are shown to be simply represented in normalized form for such applications. They are subsequently used for comparison with the dynamic behavior of resonators.

For crystal resonators subjected to wide temperature variations, the angle of cut is ordinarily chosen to minimize the frequency excursions over the entire temperature range. High precision oscillators, on the other hand, utilize resonators maintained within an oven system that closely regulates the thermal environment. In this latter case it is important to know how the unavoidable oven instabilities affect the resonator frequency and how the oven parameters interact with the f-T curve as a function of the crystal cut.

Use of the static f-T curve in conjunction with the known characteristics of commercial ovens leads to the conclusion that frequency stabilities orders of magnitude better than those observed ought to be possible with AT-cut resonators. The discrepancy is due to the neglect of the dynamic f-T behavior of the crystal. By making use of various types of dynamic f-T data from the literature, a phenomenological parameter can be extracted that satisfactorily explains the effects found in practice. When applied to simulations of oven cycling, it shows the dominance of dynamic effects in high stability situations when AT cuts are used.

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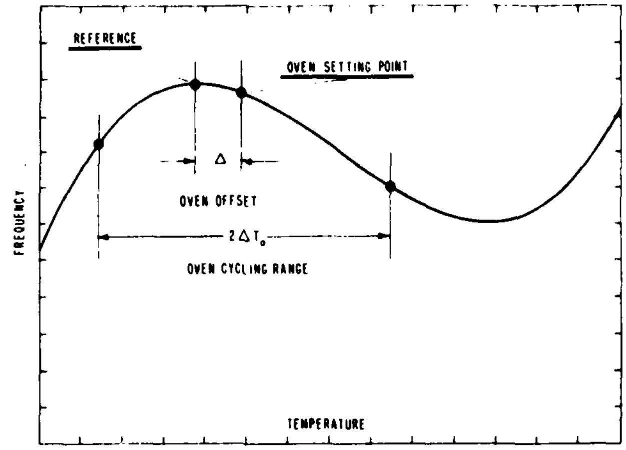

Fig. 1. Definition of oven characteristics on static frequency-temperature resonator curve. Cycling range is symmetric about setting point.

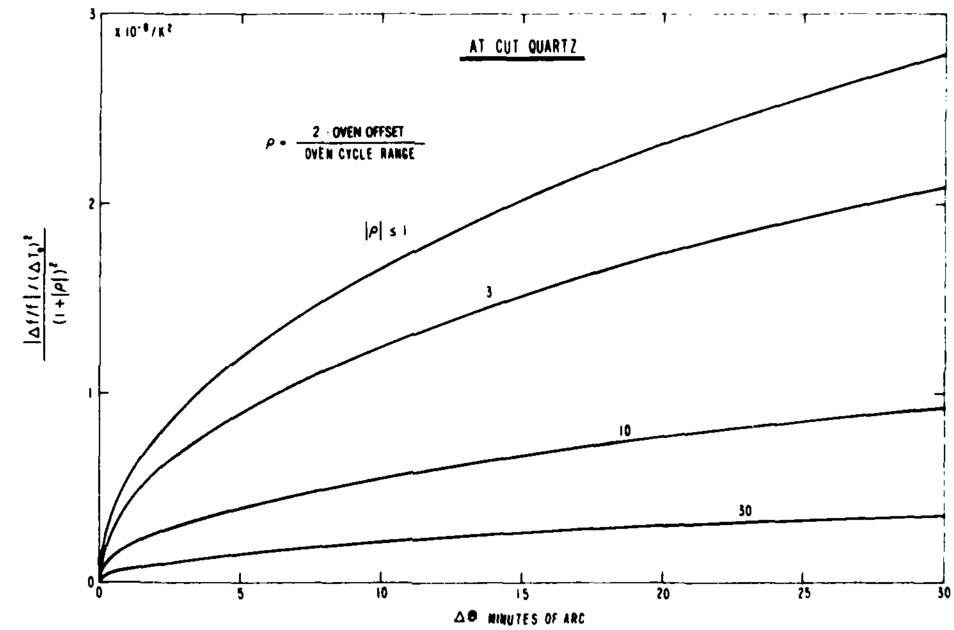

Fig. 2. Normalized frequency excursion versus orientation angle difference for *AT*-cut quartz as function of oven parameters. Abscissa reference angle is that for which the slope at the inflection point of the static frequency-temperature curve is zero. Angle varies somewhat with resonator design and construction.

At the *SC* cut the parameter linking dynamic thermal behavior to frequency becomes zero, and for this doubly rotated quartz cut the *f-T* behavior predicted from the static curve is capable of being realized.

In the first part of this paper the static *f-T* behavior of *AT* and *SC* cuts is examined. The dynamic behavior of resonators is considered in the second portion.

Static Behavior

For most types of quartz resonators the frequency-temperature behavior takes the form of the cubic curve depicted in Fig. 1. When the resonator is operated in an oven there will be, in general, an oven offset error Δ that measures the amount by which the oven setting point misses the desired reference point where the first-order temperature coefficient vanishes. In addition, the oven will have a cycling range $\pm\Delta T_0$ about the setting point, as seen in Fig. 1. As the oven cycles, the frequency will vary, but in the case shown the maximum frequency excursion will be from the reference point to the frequency at the point $\Delta + \Delta T_0$. As Δ and ΔT_0 are allowed to vary, the maximum frequency excursion will of course vary and will be a function of Δ, ΔT_0, and the shape of the cubic curve. The cubic curve varies with the angle of the cut. For the *AT* and *SC* cuts of orientations $(YXl)\theta \simeq 35.2°$, $(YXwl)\phi \simeq 21.9°/\theta \simeq 33.9°$, the reference angle is taken as that for which turnover temperatures coincide with the inflection temperature. In terms of deviations $\Delta\theta$ from this angle, the maximum frequency excursions are shown for *AT*-cut quartz in Fig. 2.

TABLE I
FREQUENCY DEVIATIONS FOR *AT*-CUT QUARTZ, WITH $\Delta\theta = 5'$ AND TRANSIENT EFFECTS OMITTED

$\Delta\theta = 5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	4.6×10^{-8}	4.6×10^{-9}	4.6×10^{-10}	4.6×10^{-11}
0.1	1.4×10^{-8}	4.7×10^{-10}	4.7×10^{-11}	4.7×10^{-12}
0.01	1.2×10^{-8}	1.4×10^{-10}	4.7×10^{-12}	4.7×10^{-13}
0.001	1.2×10^{-8}	1.2×10^{-10}	1.4×10^{-12}	4.7×10^{-14}
0	1.2×10^{-8}	1.2×10^{-10}	1.2×10^{-12}	1.2×10^{-14}

TABLE II
FREQUENCY DEVIATIONS FOR *AT*-CUT QUARTZ, WITH $\Delta\theta = 0.5'$ AND TRANSIENT EFFECTS OMITTED

$\Delta\theta = .5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	1.4×10^{-8}	1.4×10^{-9}	1.4×10^{-10}	1.4×10^{-11}
0.1	4.4×10^{-9}	1.5×10^{-10}	1.5×10^{-11}	1.5×10^{-12}
0.01	3.6×10^{-9}	4.5×10^{-11}	1.5×10^{-12}	1.5×10^{-13}
0.001	3.8×10^{-9}	3.8×10^{-11}	4.5×10^{-13}	1.5×10^{-14}
0	3.8×10^{-9}	3.8×10^{-11}	3.7×10^{-13}	3.7×10^{-15}

The angle difference is in minutes of arc, and the ordinate is the quantity

$$|\Delta f/f|_{max}/\{(\Delta T_0)^2\cdot(1+|\rho|)^2\}. \qquad (1)$$

Rho is defined as

$$\rho = \Delta/\Delta T_0 = 2\cdot\text{offset/cycle range}. \qquad (2)$$

For $|\rho| \leq 1$, a single curve suffices. The curves may be approximated by the relation

$$O \simeq (K_1 + K_2\,\Delta\theta)\cdot(\Delta\theta)^{1/2}, \qquad (3)$$

where O is the ordinate expressed in (1) and K_1, K_2 are simple functions of $|\rho|$ for a given cut. Tables I and II provide values of $|\Delta f/f|_{max}$ for *AT*-cut quartz for $\Delta\theta = 5$ and 0.5 minutes of arc, respectively.

The doubly rotated *SC* cut of orientation $(YXwl)\phi \simeq 21.9°/\theta \simeq +33.9°$ has two angles to vary. Fig. 3 gives the variation in O versus $\Delta\theta$. Tables III and IV present values of $|\Delta f/f|_{max}$ for $\Delta\theta$ and $\Delta\phi = 5'$, respectively, while Tables V and VI are for $\Delta\theta$ and $\Delta\phi = 0.5'$, also respectively.

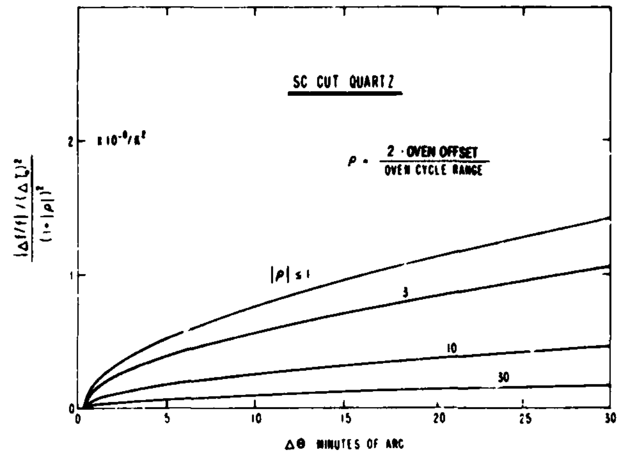

Fig. 3. Normalized frequency excursion versus orientation angle difference for *SC*-cut quartz as function of oven parameters. Curves are weak functions of azimuthal angle ϕ.

TABLE III
FREQUENCY DEVIATIONS FOR *SC*-CUT QUARTZ, WITH $\Delta\theta = 5'$

$\Delta\theta = 5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	2.1×10^{-8}	2.1×10^{-9}	2.1×10^{-10}	2.1×10^{-11}
0.1	6.5×10^{-9}	2.2×10^{-10}	2.2×10^{-11}	2.2×10^{-12}
0.01	5.4×10^{-9}	6.5×10^{-11}	2.2×10^{-12}	2.2×10^{-13}
0.001	5.4×10^{-9}	5.5×10^{-11}	6.5×10^{-13}	2.2×10^{-14}
0	5.4×10^{-9}	5.4×10^{-11}	5.4×10^{-13}	5.4×10^{-15}

TABLE IV
FREQUENCY DEVIATIONS FOR *SC*-CUT QUARTZ, WITH $\Delta\phi = 5'$

$\Delta\phi = 5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	5.9×10^{-9}	6.0×10^{-10}	6.0×10^{-11}	6.0×10^{-12}
0.1	1.8×10^{-9}	6.3×10^{-11}	6.3×10^{-12}	6.3×10^{-13}
0.01	1.6×10^{-9}	1.9×10^{-11}	6.3×10^{-13}	6.3×10^{-14}
0.001	1.6×10^{-9}	1.6×10^{-11}	1.9×10^{-13}	6.3×10^{-15}
0	1.6×10^{-9}	1.6×10^{-11}	1.6×10^{-13}	1.6×10^{-15}

With reference to Table II, one sees that an *AT* cut operated in a good oven with a stability of a few millikelvins should produce frequency stabilities in the order 10^{-13} to 10^{-14}. This in fact is far from the actual values obtained.

TABLE V
FREQUENCY DEVIATIONS FOR *SC*-CUT QUARTZ, WITH $\Delta\theta = 0.5'$

$\Delta\theta = .5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	6.3×10^{-9}	6.4×10^{-10}	6.4×10^{-11}	6.4×10^{-12}
0.1	2.0×10^{-9}	6.7×10^{-11}	6.7×10^{-12}	6.7×10^{-13}
0.01	1.7×10^{-9}	2.0×10^{-11}	6.7×10^{-13}	6.7×10^{-14}
0.001	1.7×10^{-9}	1.7×10^{-11}	2.0×10^{-13}	6.7×10^{-15}
0	1.7×10^{-9}	1.7×10^{-11}	1.7×10^{-13}	1.7×10^{-15}

TABLE VI
FREQUENCY DEVIATIONS FOR *SC*-CUT QUARTZ, WITH $\Delta\phi = 0.5'$

$\Delta\phi = .5'$	OVEN CYCLING RANGE (K)			
OVEN OFFSET (K)	1	0.1	0.01	0.001
1	1.6×10^{-9}	1.7×10^{-10}	1.7×10^{-11}	1.7×10^{-12}
0.1	5.4×10^{-10}	2.0×10^{-11}	2.0×10^{-12}	2.0×10^{-13}
0.01	5.4×10^{-10}	6.0×10^{-12}	2.0×10^{-13}	2.0×10^{-14}
0.001	5.5×10^{-10}	5.1×10^{-12}	6.1×10^{-14}	2.0×10^{-15}
0	5.5×10^{-10}	5.1×10^{-12}	5.0×10^{-14}	5.0×10^{-16}

DYNAMIC BEHAVIOR

That the resonance frequency of a crystal vibrator depends on temperature-rate and gradient effects, as well as temperature itself, has been known for some time [3]-[26]. Experimental evidence exists in a number of forms. Bistline [13] showed that the f-T cubic curve appeared to exhibit cut angles differing in $\Delta\theta$ by several minutes of arc for different temperature sweep rates. Kusters [20] showed how the upper turning point of an AT cut shifts in frequency with temperature rate. In both cases the temperature-time graphs were ramps, so that $\dot{T} = dT/dt$ is a constant. The influence of a "step function" of temperature on AT cuts was shown by Warner [7], [11] and Munn [14]. For BT-cut and rotated-X-cut crystals [27], the frequency spike is reversed in sign. Based upon the observed effects, a number of models have been used to characterize the thermal transient, or dynamic thermal, behavior of quartz resonators [9], [16], [19], [21], [24].

We propose here a simple model that explains the results of a variety of experiments. More careful experiments, coupled with a thorough analysis involving nonlinear elasticity theory, are required to provide a firm foundation for understanding dynamic effects in resonators. It is hoped that the results presented here will stimulate such an approach to the problem. The traditional manner of expressing the static frequency-temperature behavior of quartz resonators is due to Bechmann [28]:

$$\Delta f/f = a_0\,\Delta T + b_0\,\Delta T^2 + c_0\,\Delta T^3. \tag{4}$$

In (4) $\Delta T = T - T_0$, where T is the present temperature and T_0 is the specified reference temperature, often taken as 25°C.

The parameters a_0, b_0, and c_0 depend on material, cut, geometry, electroding, etc., but do not depend on time, for a given resonator design. Our assumption consists of the inclusion of the term $\tilde{a}\cdot\dot{T}$ in the expression for the effective first-order temperature coefficient $a(t)$, which now depends on time implicitly through the temporal behavior of the temperature:

$$\Delta f(t)/f = a(t)\cdot\Delta T(t) + b_0\cdot\Delta T(t)^2 + c_0\cdot\Delta T(t)^3, \tag{5}$$

with

$$a(t) = a_0 + \tilde{a}\cdot\dot{T}(t). \tag{6}$$

The quantity $\tilde{a}$ is a function of the same entities as a_0, b_0, and c_0, as well as several others such as the method of setting up thermal gradients (thickness gradient, azimuth-dependent lateral gradients), thermal conductivity of supports and electrodes, etc., but is a constant for a given design. It is treated simply as a phenomenological quantity to be evaluated separately for each crystal design. It is obvious that the assumption concerning $a(t)$ could be extended to the coefficients b_0 and c_0 by introducing similar constants $\tilde{b}$ and $\tilde{c}$; the results are qualitatively the same, and they will be omitted here.

PUBLISHED EXPERIMENTS

For the case of a ramp temperature-time profile, where $\dot{T}$ is constant, $a(t)$ will reduce to a constant different from a_0. The f-T curve simply appears as if it had changed its apparent orientation angle θ. In the case of Bistline's graphs, the value of $\tilde{a}$ required to account for the data is $+1.7\times10^{-7}$ s/K². This value is arrived at using the known quantity $da/d\theta = -5.08\times10^{-6}$/K, deg θ for the AT cut [2], [23].

Applying the model to Kusters' curves [20], straightforward algebra yields the relation for the normalized frequency shift of the turning point

$$\delta = (T_\mu - T_0)\cdot\tilde{a}\cdot\dot{T}, \tag{7}$$

where T_μ is the upper turnover temperature and T_0 is the reference temperature. Using the parameters of the AT-cut crystal discussed by Kusters [20], the value of $\tilde{a}$, for the crystal used by him, is

$$\tilde{a} = -2.27\times10^{-7}\quad \text{s/K}^2. \tag{8}$$

The Warner curves [7], [11], [14] may be simulated closely by making the assumption that the thermal "step function" is of the form $(1 - e^{-t/\tau})$ and involves a single system thermal time constant τ. The values of $\tilde{a}$ are larger in magnitude than for the Bistline and Kusters experiments. The sign of $\tilde{a}$ changes according to the electrode metallization pattern, being negative for thickness excitation and positive for electrodes with a gap for lateral excitation [11]. It may be expected that other factors will influence the sign and size of $\tilde{a}$ and that a proper

choice of resonator and electrode design, electrode material(s), and method of deposition will produce resonators with small $\tilde{a}$ values. The possibility of controlling $\tilde{a}$ by the use of doubly rotated plates is discussed below.

The presence of a frequency spike, as found by Warner in response to a thermal step function applied to an *AT*-cut bulk wave resonator, has been recently verified for surface acoustic wave (SAW) resonators of *ST*-cut orientation by Parker [29].

Sinusoidal Temperature Variation

Based upon the simple model adopted it is possible to simulate the effect of sweeping the temperature in a sinusoidal manner. It is known that this leads experimentally to hysteresis effects. The result of a wide temperature range sweep is shown in Fig. 4. The heavy line represents the static f-T curve. When the temperature is swept as $\sin \omega t$ by 60K about the 30°C point, the resulting curve exhibits an hysteresis that varies with the rate ω as shown. The effect is more pronounced at the upper turning point (UTP); this is due to the sign of $\tilde{a}$. The orbits are counterclockwise, also because of the sign of $\tilde{a}$, at the UTP.

The general features of the curves in Fig. 4, representing dynamic thermal cyclings, have been verified in the main by experiments performed by Hughes [30] using sinusoidal temperature cyclings. In particular, the much greater deviations at the UTP (implying a negative $\tilde{a}$ in his experiment), compared with the lower turning point, were unequivocally established for the resonator examined.

Fig. 5 shows the hysteresis effects expected for ±10K excursions about the UTP. For a sweep with a 10-min period the frequency change is roughly three times that expected on the basis of the static curve. In Fig. 6 hysteresis orbits for $\Delta T_0 = 5$K, offset from the UTP at T_μ by +5K and by −10K, are shown.

The frequency scale is magnified in Fig. 7 to parts in 10^{-10}. For temperature deviations ΔT_0 of $\pm 5 \times 10^{-3}$ K, the static f-T curve is nearly a horizontal line, while the orbits for the dynamic frequency behavior are elliptical, with amplitudes greatly exceeding the static behavior when the period of the temperature cycle is shorter than about 1 h. The constant $\tilde{a}$ from (8) has been used in the computations. In Fig. 8 the frequency scale has been further magnified to the 10^{-12} range, with a temperature scale in millikelvins about the UTP. Again we can see the dominance of the dynamic behavior compared to the static, even when the temperature variations take place over periods of the order of a day.

Table VII provides values for the fractional frequency deviation $|\Delta f/f|_{max}$ due to the dynamic thermal effect in *AT*-cut quartz, as a function of both oven cycling range and cycling time, assuming sinusoidal temperature variation and the $\tilde{a}$ determined from Kusters' experiment [20]. The oven offset has been taken as zero. The entries are to be compared to those in Tables I and II. The dramatic influence of the dynamic component of the frequency shift can be seen from the comparison.

Gagnepain [31] has confirmed the hysteresis orbits shown, e.g., in Figs. 5–8, for cyclings in a small temperature range near a turnover point; he uses a model slightly different from that of (5) and (6), where the $\tilde{a}$ term is not multiplied by $\Delta T(t)$ in (5); this model has been used by Anderson and Merrill (9) and

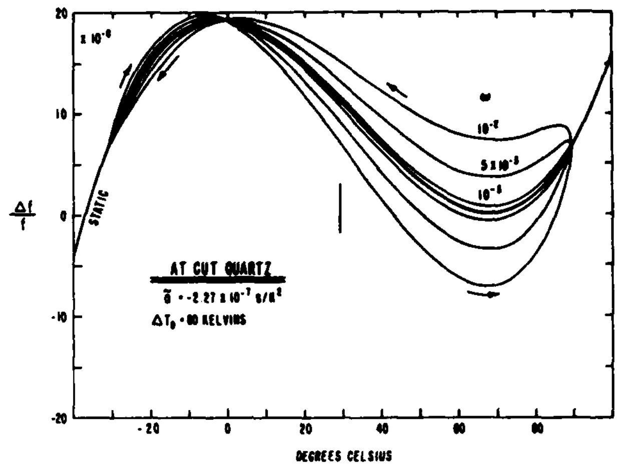

Fig. 4. Simulation of frequency hysteresis arising from sinusoidal temperature cycling. Sixty-kelvin sweep about inflection temperature. Orbital periods are 628, 1257, and 6283 s for $\omega = 10^{-2}, 5 \times 10^{-3}$, and 10^{-3}, respectively.

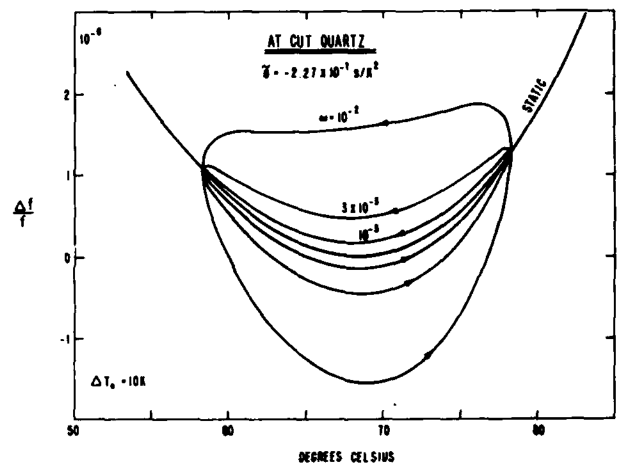

Fig. 5. Simulation of frequency hysteresis arising from sinusoidal temperature cycling. Ten kelvin sweep about upper turnover point. Counterclockwise orbits stem from sign of $\tilde{a}$.

Koehler *et al.* [21], [24]. For small temperature sweeps both models give equivalent results, but for wide sweeps (5) predicts an apparent orientation angle shift for temperature ramp excitation, whereas the modified model predicts a constant frequency offset. Experimental results for ramp functions of temperature have been obtained by Lagasse [32], whose curves show very interesting behavior. Data were supplied on two groups of *AT*-cut resonators having thicknesses very nearly in the ratio of 2:1. For the thinner plates, 60% showed only a constant frequency offset, while the remainder showed both an offset as well as a rotation in the apparent angle. The thicker plates showed the frequency offset alone in only 10% of the cases, while 90% had both an offset and an apparent angle change. Each resonator in a group was of identical construction, but between the groups the designs were similar but not identical.

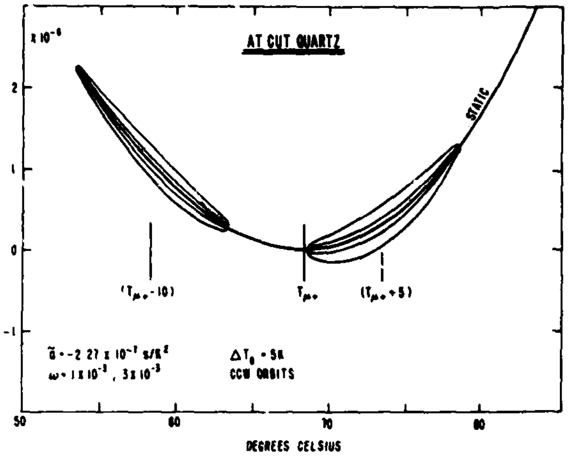

Fig. 6. Hysteresis loops for sinusoidal temperature cycles offset from upper turnover point. Cycling range is five kelvins; offsets are −10 and +5 K. Orbital periods are 2094 s for outer loops and 6283 s for inner loops, corresponding to $\omega = 3 \times 10^{-3}$ and 10^{-3}, respectively.

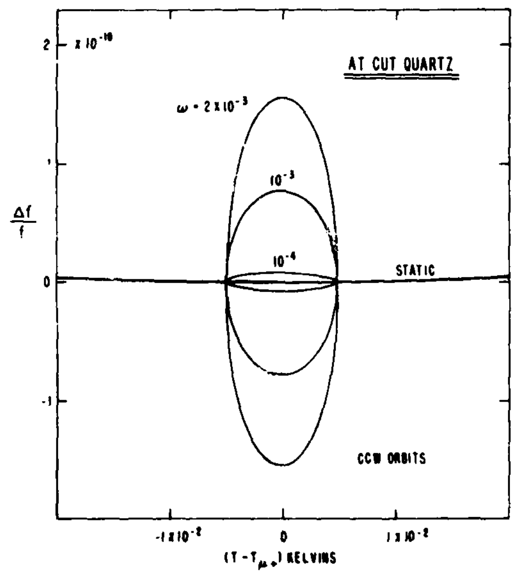

Fig. 7. Elliptical hysteresis orbits for sinusoidal temperature cycling. Temperature sweep range 5 millikelvins. This figure is drawn for $\tilde{a} = -2.27 \times 10^{-7}$ s/K^2. Orbital periods are 3142, 6283, and 62832 s, corresponding to $\omega = 2 \times 10^{-3}, 10^{-3}$, and 10^{-4}, respectively.

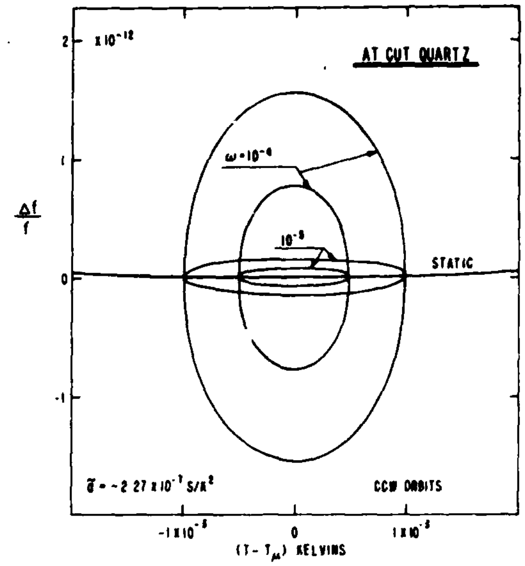

Fig. 8. Elliptical hysteresis orbits for sinusoidal temperature cycling. Temperature sweep ranges 0.5 and 1 millikelvin. Orbital periods are 6.28×10^4 and 6.28×10^5 s, for $\omega = 10^{-4}$ and 10^{-5}, respectively. It is seen that even diurnal rhythms contribute significantly to frequency instability.

TABLE VII
FREQUENCY DEVIATIONS DUE TO DYNAMIC THERMAL BEHAVIOR OF *AT*-CUT RESONATOR OF [20]

$\Delta = 0$	OVEN CYCLING RANGE (K)			
	1	0.1	0.01	0.001
OVEN CYCLING TIME: 10 SEC.	9.8×10^{-6}	9.8×10^{-7}	9.8×10^{-8}	9.8×10^{-9}
1 MIN.	1.6×10^{-6}	1.6×10^{-7}	1.6×10^{-8}	1.6×10^{-9}
1 HOUR	2.7×10^{-8}	2.7×10^{-9}	2.7×10^{-10}	2.7×10^{-11}
1 DAY	1.1×10^{-9}	1.1×10^{-10}	1.1×10^{-11}	1.1×10^{-12}

The simulations seen in Figs. 4-8 are for sinusoidal variations in temperature. In practice, the fluctuations in the thermal field will have a certain spectral distribution. The result of the coupling term $\tilde{a}$ will be to produce a corresponding frequency spectral distribution, with a resulting magnified frequency instability when compared to what would be expected in the static case.

What are the prospects for reducing $\tilde{a}$? Fortunately, it appears that by design even *AT*-cut resonators can be produced with smaller values of $\tilde{a}$. However, by using doubly rotated crystals cut to orientations in the vicinity of the *SC* cut the value of $\tilde{a}$ can be reduced to a negligible value. These cuts are compensated for a variety of other effects as well [23], [33], [34]. For rotated-*X*-cuts on the zero temperature coefficient locus [35] the approximate value for $\tilde{a}$ is $+1.8 \times 10^{-7}$ s/K^2 for EerNisse's design. For a given vibrator design, a plot of $-\tilde{a}$ versus ϕ along the upper zero temperature coefficient locus in quartz would look very much like Fig. 3 in [33]; the derivative $\partial\tilde{a}/\partial\phi$ is zero at the *AT* cut and reaches its maximum positive value at the rotated-*X*-cut.

The *SC* cut permits the possibility of a revolutionary change in the stabilities of crystal oscillators, particularly in the intermediate-to-long-term regimes. With the coefficient $\tilde{a}$ reduced to a negligible level, and with state-of-the-art ovens in the millikelvin range, stabilities in the range 10^{-13} to 10^{-14} ought to be achievable. A stability of 6×10^{-14} for 128-s sampling times has recently been achieved with an *SC*-cut crystal of special design, utilizing an airgap fixture and special mountings [36], [37]. This stability also depended upon a special circuit configuration in addition to the *SC*-cut resonator [38]. The ability to achieve medium-term stabilities on this order would make doubly rotated quartz resonators com-

parable to rubidium atomic frequency standards in precision applications. Medium precision stabilities will be available for fast warmup oscillators, and they represent a second important application area for this latest representative of a long line of quartz vibrators.

Conclusion

The actual frequency instabilities observed in quartz crystal oscillators for medium- and high-precision applications are greater than can be accounted for by the static frequency-temperature characteristic of the vibrator and the noise characteristics of the active devices and ancillary circuitry. One mechanism contributing to the instability is the dynamic f-T coupling. This paper has used available experimental data with a simple mathematical model to explore some of the consequences of the dynamic effect on the characteristics of the crystal resonance frequency.

Doubly rotated quartz resonators that are compensated for the dynamic thermal effect offer promise in the near-term future of making available very small, light weight, and cheap oscillators with stabilities approaching those of rubidium frequency standards.

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Note: Many of the references were presented at the Annual Frequency Control Symposium, U.S. Army Electronics R&D Command, Fort Monmouth, NJ 07703. They are cited here as AFCS for brevity.

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